**Unit 2 - Reading 2**

**Number Lines, Distance, Displacement,** **Average Speed, and Average Velocity**

Adapted from AMTA 2006

The terms speed and velocity are terms that are commonly used when describing motion. However, we really have no way to measure either of these quantities directly. In order to describe any of these quantities, one must begin with more fundamental quantities. Specifically, we must measure position, which is where an object is located, and time, which is when the object is at a particular location. By considering where something is, when it is there, and tracking the position of an object as time rolls on, one can determine the values of the an object’s average speed and average velocity.

**Distance, Position, and Displacement**

|  |  |  |
| --- | --- | --- |
| **Name** | **Description** | **Symbol** |
| ***Distance*** | How far something has traveled along some path |  |
| ***Position*** | Where something is located in some reference system. |  |
| ***Displacement*** | The difference between an object’s starting position and its ending position. |  |

To illustrate the difference between these three quantities, consider a number line calibrated in meters. Assume that each numbered mark on the number line below is 1 meter from the one next to it. Let us also consider that we will call to the right the positive direction and to the left the negative direction.

0

1

2

3

4

5

6

7

8

9

10

+

(meters)

Now consider an object that takes a trip along the number line, beginning at a position of +2 meters, and ending at a position of +7 meters.

0

1

2

3

4

5

6

7

8

9

10

+

(meters)

Trip 1

How would we describe the distance, position(s), and displacement associated with this trip?

Clearly, we must consider at least two positions—the starting position and the ending position, even though the object occupied many positions during its trip from the +2 meter position to the +7 meter position.

The object has a ***starting position*** of +2 meters.

The object has and ***ending position*** of +7 meters.

The object traveled a ***distance*** along the number line of 5 meters.

The object had a ***displacement*** of 5 meters in the positive direction or +5 meters.

What if the object reversed its trip, starting at a position of +7 meters and ending at a position of +2 meters? Which of the quantities above have changed as a result of this new trip?

0

1

2

3

4

5

6

7

8

9

10

+

(meters)

Trip 2

The object has a ***starting position*** of +7 meters.

The object has and ***ending position*** of +2 meters.

The object traveled a ***distance*** along the number line of 5 meters.

The object had a ***displacement*** of 5 meters in the negative direction or -5 meters.

So you can see that while distance tells you only how far something has traveled, displacement tells you how far it is from the starting point to the ending point as well as which direction you would have to travel to go from the starting point to the ending point. For the first trip the distance traveled was 5 meters, and the same was true for the second trip. The displacements, however, were quite different, because the object traveled in two different directions for the two trips.

Finally, let us consider a more complicated trip along the number line in order to more fully illustrate the difference between distance and displacement. Consider an object that travels from a position of +2 meters to a position of +9 meters and then turns around and goes back to a position of +7 meters.

0

1

2

3

4

5

6

7

8

9

10

+

(meters)

Trip 3

Let’s consider the positions, distance and displacement for this trip.

The object has a ***starting position*** of +2 meters.

The object has an ***intermediate position*** of +9 meters, which is where it changed directions.

The object has and ***ending position*** of +7 meters.

The object traveled a ***distance*** along the number line of 9 meters.

In this example, the object traveled from a position of +2 meters to a position of +9 meters, a distance of 7 meters. It then traveled from a position of +9 meters to a position of +7 meters, a distance of 2 meters. If it traveled 7 meters and then 2 meters, its total distance traveled would be 9 meters.

The object had a ***displacement*** of 5 meters in the positive direction or +5 meters.

Even though the object traveled a considerably greater distance in this trip compared to the first trip (9 meters compared to 5 meters), the change in position or displacement was exactly the same: 5 meters in the positive direction. This is because the starting and ending positions are exactly the same for this trip as they were for the first trip. So you can see that distance takes into consideration the total amount traveled along a given path, where as displacement is concerned only with where you started, where you ended, and which direction you traveled.

## **Speed and Velocity**

This also brings us to the differences between speed and velocity. Let us assume that each of the three trips required 5 seconds to complete. What would the object’s average speed and average velocity be for each trip based on the mathematical definitions of average speed and average velocity.

The common sense notion of speed is that it measures how fast something travels. This notion is correct. The average speed of an object is determined by dividing the distance traveled by the object by the time elapsed while the object was traveling that distance.

Expressed as an equation,

|  |  |
| --- | --- |
| Average speed = | distance traveled |
| elapsed time |

The symbol that we will use for average speed is .

Using this symbol for average speed, *d* for distance traveled, and ∆*t* for elapsed time,

the equation for average speed would be expressed symbolically by

Thus we arrive at a measure, for a given trip by an object, of its average rate of travel (average speed) measured in some distance unit divided by some time unit. In the U.S. most speed limits are measured in the unit miles per hour. In Mexico they speed limit signs are in kilometers per hour. But speed can be measured in any distance unit divided by any time unit. Other examples could be meters per second, feet per second, or some outrageous unit like furlongs per fortnight or millimeters per millennium. Average speed tells us about the rate of travel, but tells us nothing about the direction of travel.

To know both the rate of travel and the direction of motion, we rely on a quantity called velocity. In order to specify an object’s velocity, we must describe both how fast it is traveling (its speed) and the direction in which it travels. Velocity is defined as the rate at which an object changes its position with respect to time.

Expressed as an equation,

The symbol that we will use for average velocity is .

Using this symbol for average velocity, for change in position (displacement), and for elapsed time, the equation for average velocity would be expressed symbolically as:

A reasonable question might be: How is average velocity different from average speed? The difference lies in the difference between distance and displacement.

In the first trip the object traveled 5 meters in 5 seconds. Since average speed is calculated by dividing the distance traveled by the elapsed time, this object had an average speed of 5 meters divided by 5 seconds which yields 1 meter per second. Try calculating the average speed for the object in trip 2. Did you get 1 meter per second for trip 2? (It also traveled 5 meters in 5 seconds) What did you get for trip 3? Since the object traveled 9 meters in 5 seconds, the average speed would have to be 9 meters divided by 5 seconds which brings its average speed to 9/5 meters per second, or in decimal form, 1.8 meters per second.

What about average velocity? In trip 1 the object had a change of position of 5 meters in the positive direction. The object therefore had an average velocity of +5 m divided by 5 seconds. Its average velocity is therefore 1 meter per second in the positive direction. Do you see that average velocity gives us an indication of the direction the object traveled as well as how fast (on the average) the object traveled?

For trip 2, the object changed its position by 5 meters in the negative direction in a time of 5 seconds. This object had an average velocity of –5 m divided by 5 seconds. This yields -1 meter per second or 1 meter per second in the negative direction. The sign on the velocity tells us the direction the object traveled, just like the sign on the displacement told us the direction it traveled.

Finally, for trip 3, the object changed its position by 5 meters in the positive direction in a time of 5 seconds. This object had exactly the same average velocity as the object in trip 1, even though the actual trips were quite different. Both objects changed their position by 5 meters in the positive direction in a time of 5 s. They both had an average velocity of 1 meter per second in the positive direction or +1 meters per second. Notice that they did not have the same average speed. This is because the object in trip 3 covered more total ground in its 5 seconds of travel than did the object in trip 1. They had the same average velocity because they both changed their positions by the same amount (+5 meters) in the same amount of time (5 seconds).